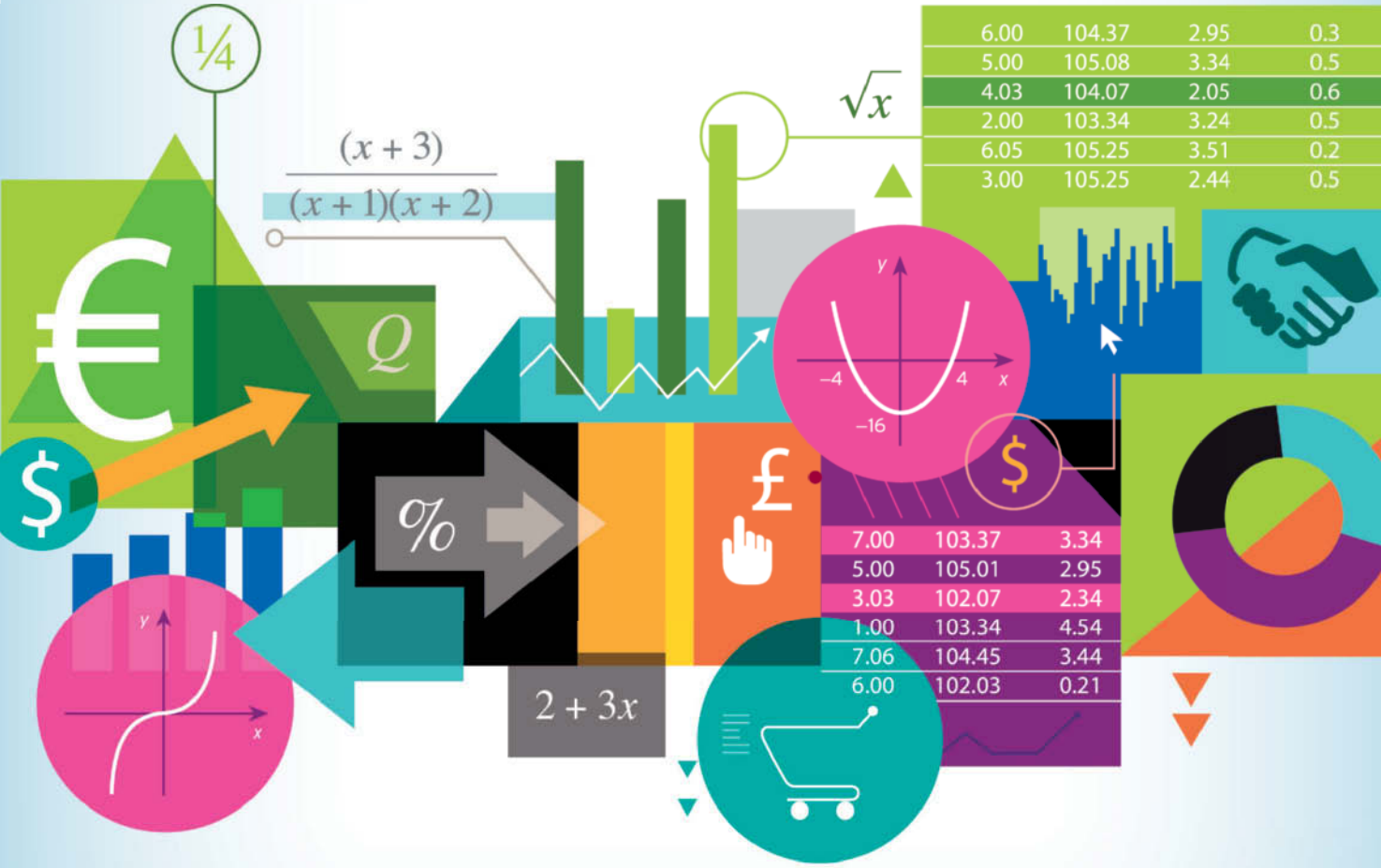
MATHEMATICS

FOR ECONOMICS AND BUSINESS

IAN JACQUES



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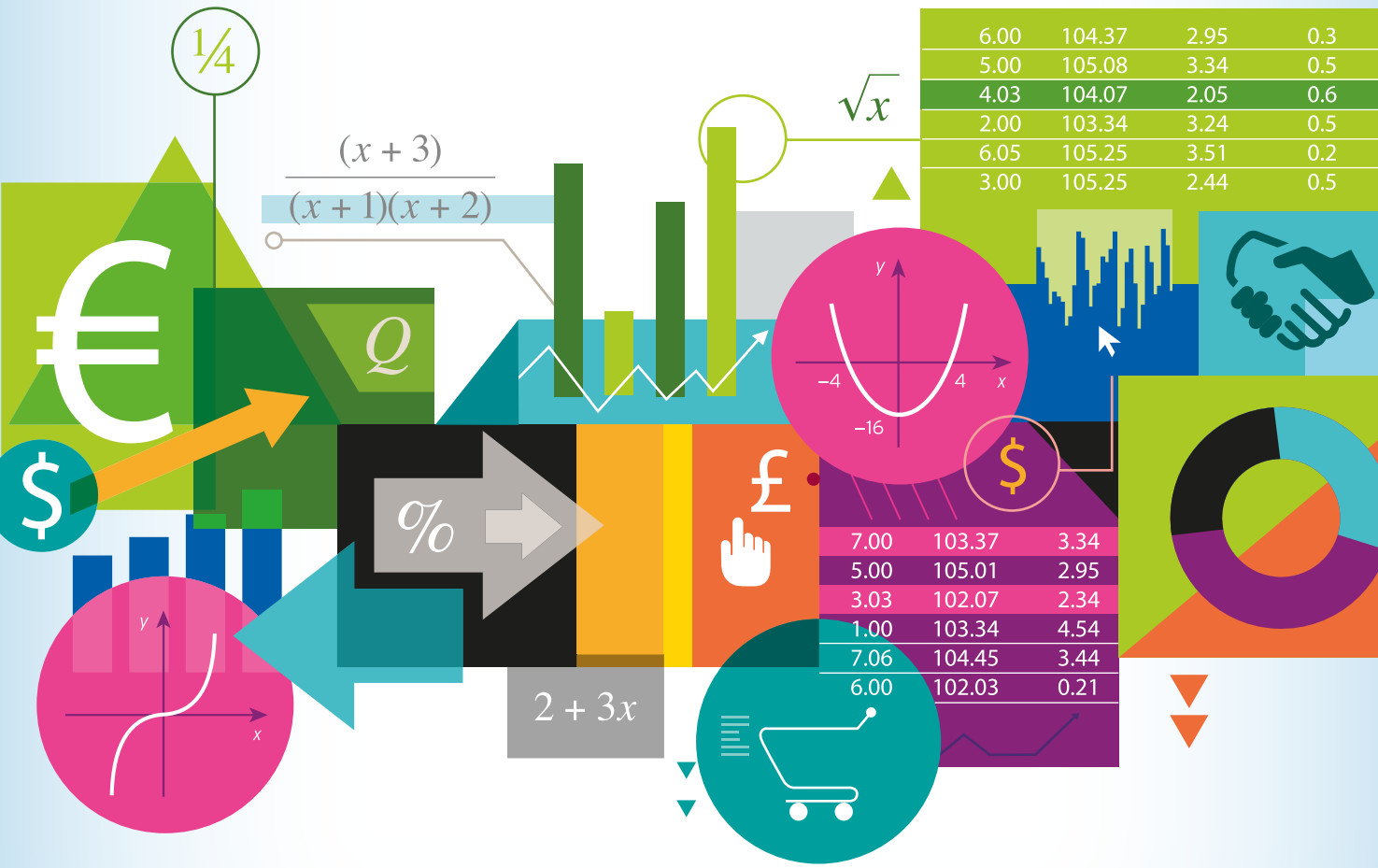

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PEARSON EDUCATION LIMITED
KAO Two
KAO Park
Harlow CM17 9NA
United Kingdom
Tel: +44 (0)1279 623623
Web: www.pearson.com/uk

First published 1991 (print)
Second edition published 1994 (print)
Third edition published 1999 (print)
Fourth edition published 2003 (print)
Fifth edition published 2006 (print)
Sixth edition published 2009 (print)
Seventh edition published 2013 (print and electronic)
Eighth edition published 2015 (print and electronic)
Ninth edition published 2018 (print and electronic)

ISBN: 978-1-292-19166-9 (print)
978-1-292-19170-6 (PDF)
978-1-292-19171-3 (ePub)

British Library Cataloguing-in-Publication Data
A catalogue record for the print edition is available from the British Library

Library of Congress Cataloging-in-Publication Data
Names: Jacques, Ian, 1957– author.
Title: Mathematics for economics and business / Ian Jacques.
Description: Ninth edition. | Harlow, United Kingdom : Pearson Education, [2018] | Includes bibliographical references and index.
Identifiers: LCCN 2017049617| ISBN 9781292191669 (print) | ISBN 9781292191706 (pdf) | ISBN 9781292191713 (epub)
Subjects: LCSH: Economics, Mathematical. | Business mathematics.
Classification: LCC HB135 .J32 2018 | DDC 512.024/33—dc23
LC record available at https://lccn.loc.gov/2017049617

10 9 8 7 6 5 4 3 2 1
22 21 20 19 18

Front cover image © Getty Images

Print edition typeset in 10/12.5pt Sabon MT Pro by iEnergizer Aptara®, Ltd.
Printed in Slovakia by Neografia

NOTE THAT ANY PAGE CROSS REFERENCES REFER TO THE PRINT EDITION

To Victoria, Lewis and Celia

CONTENTS

PREFACE

This text is intended primarily for students on economics, business studies and management courses. It assumes very little prerequisite knowledge, so it can be read by students who have not undertaken a mathematics course for some time. The style is informal, and the text contains a large number of worked examples. Students are encouraged to tackle problems for themselves as they read through each section. Detailed solutions are provided so that all answers can be checked. Consequently, it should be possible to work through this text on a self-study basis. The material is wide ranging and varies from elementary topics such as percentages and linear equations to more sophisticated topics such as constrained optimisation of multivariate functions. The text should therefore be suitable for use on both low- and high-level quantitative methods courses.

This text was first published in 1991. The prime motivation for writing it then was to try to produce a text that students could actually read and understand for themselves. This remains the guiding principle when writing this ninth edition.

One of the main improvements is the inclusion of over 200 additional questions. Each chapter now ends with both multiple choice questions and a selection of longer examination-style questions. Students usually enjoy tackling multiple choice questions since they provide a quick way of testing recall of the material covered in each chapter. Several universities include multiple choice as part of their assessment. The final section in each chapter entitled "Examination Questions" contains longer problems which require knowledge and understanding of more than one topic. Although these have been conveniently placed at the end of each chapter it may be best to leave these until the end of the academic year so that they can be used during the revision period just before the examinations.

Ian Jacques

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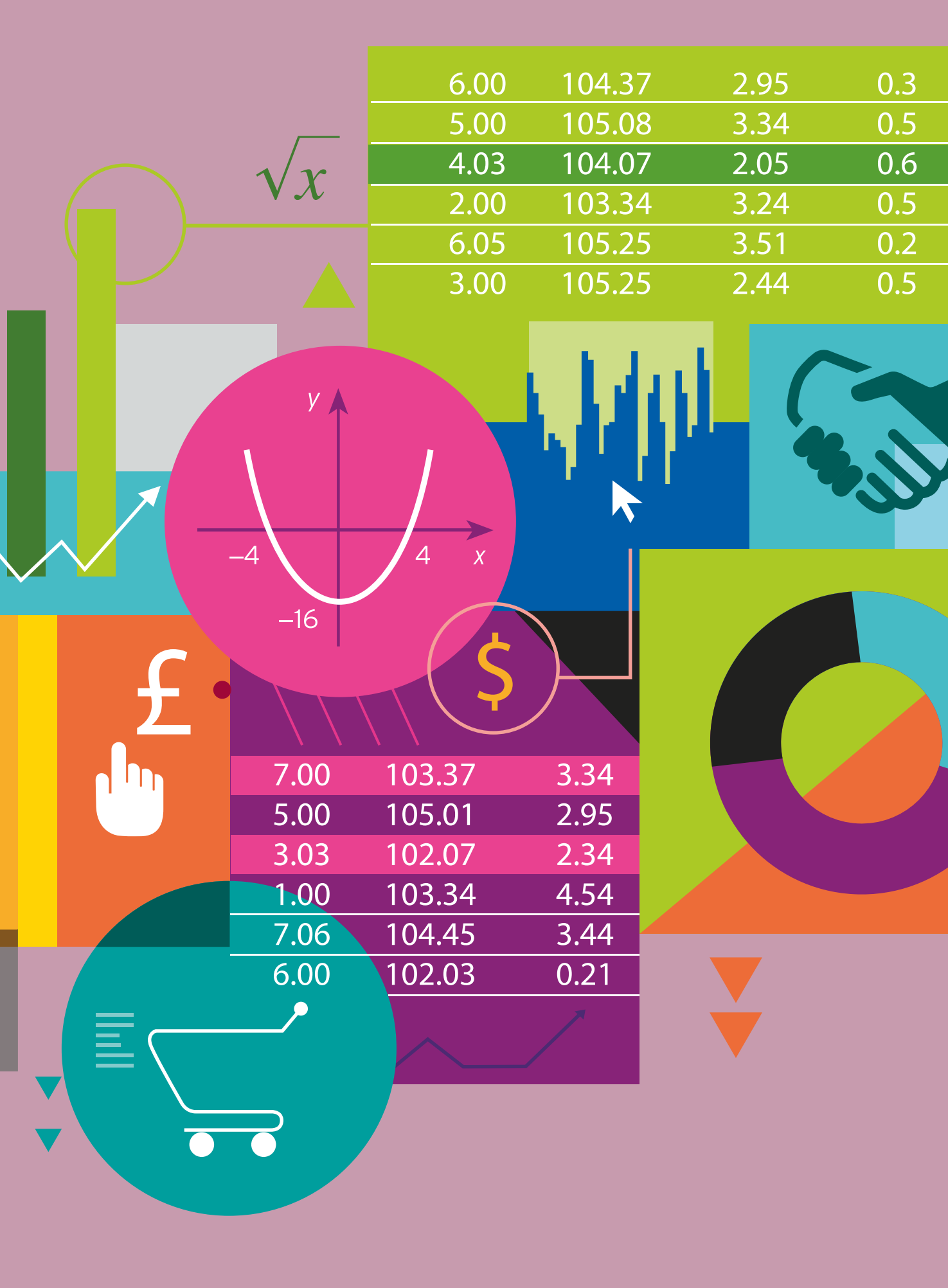
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INTRODUCTION

Getting Started

NOTES FOR STUDENTS: HOW TO USE THIS TEXT

I am always amazed by the mix of students on first-year economics courses. Some have not acquired any mathematical knowledge beyond elementary algebra (and even that can be of a rather dubious nature), some have never studied economics before in their lives, while others have passed preliminary courses in both. Whatever category you are in, I hope that you will find this text of value. The chapters covering algebraic manipulation, simple calculus, finance, matrices and linear programming should also benefit students on business studies and management courses.

The first few chapters are aimed at complete beginners and students who have not taken mathematics courses for some time. I would like to think that these students once enjoyed mathematics and had every intention of continuing their studies in this area, but somehow never found the time to fit it into an already overcrowded academic timetable. However, I suspect that the reality is rather different. Possibly they hated the subject, could not understand it and dropped it at the earliest opportunity. If you find yourself in this position, you are probably horrified to discover that you must embark on a quantitative methods course with an examination looming on the horizon. However, there is no need to worry. My experience is that every student is capable of passing a mathematics examination. All that is required is a commitment to study and a willingness to suspend any prejudices about the subject gained at school. The fact that you have bothered to buy this text at all suggests that you are prepared to do both.

To help you get the most out of this text, let me compare the working practices of economics and engineering students. The former rarely read individual books in any great depth. They tend to skim through a selection of books in the university library and perform a large number of Internet searches, picking out relevant information. Indeed, the ability to read selectively and to compare various sources of information is an important skill that all arts and social science students must acquire. Engineering students, on the other hand, are more likely to read just a few books in any one year. They read each of these from cover to cover and attempt virtually every problem en route. Even though you are most definitely not an engineer, it is the engineering approach that you need to adopt while studying mathematics. There are several reasons for this. First, a mathematics text can never be described, even by its most ardent admirers, as a good bedtime read. It can take an hour or two of concentrated effort to understand just a few pages of a mathematics text. You are therefore recommended to work through this text systematically in short bursts rather than to attempt to read whole chapters. Each section is designed to take between one and two hours to complete, and this is quite sufficient for a single session. Secondly, mathematics is a hierarchical subject in which one topic follows on from the next. A construction firm building an office block is hardly likely to erect the fiftieth storey without making sure that the intermediate floors and foundations are securely in place. Likewise, you cannot 'dip' into the middle of a mathematics text and expect to follow it unless you have satisfied the prerequisites for that topic. Finally, you actually need to do mathematics yourself before you can understand it. No matter how wonderful your lecturer is, and no matter how many problems are discussed in class, it is only

by solving problems yourself that you are ever going to become confident in using and applying mathematical techniques. For this reason, several problems are interspersed within the text, and you are encouraged to tackle these as you go along. You will require writing paper, graph paper, pens and a calculator for this. There is no need to buy an expensive calculator unless you are feeling particularly wealthy at the moment. A bottom-of-the-range **scientific** calculator should be good enough. Answers to every question are printed at the back of this text so that you can check your own answers quickly as you go along. However, please avoid the temptation to look at them until you have made an honest attempt at each one. Remember that in the future you may well have to sit down in an uncomfortable chair, in front of a blank sheet of paper, and be expected to produce solutions to examination questions of a similar type.

At the end of each section there are two parallel exercises. The non-starred exercises are intended for students who are meeting these topics for the first time and the questions are designed to consolidate basic principles. The starred exercises are more challenging but still cover the full range so that students with greater experience will be able to concentrate their efforts on these questions without having to pick-and-mix from both exercises. The chapter dependence is shown in Figure I.1. If you have studied some advanced mathematics before, you will discover that parts of Chapters 1, 2 and 4 are familiar. However, you may find that the sections on economics applications contain new material. You are best advised to test yourself by attempting a selection of problems from the starred exercise in each section to see if you need to read through it as part of a refresher course. Economics students in a desperate hurry to experience the delights of calculus can miss out Chapter 3 without any loss of continuity and move straight on to Chapter 4. The mathematics of finance is probably more relevant to business and accountancy students, although you can always read it later if it is part of your economics syllabus.

At the end of every chapter you will find a multiple choice test and some examination questions. These cover the work of the whole chapter. We recommend that you try the multiple choice questions when you have completed the relevant chapter. As usual, answers

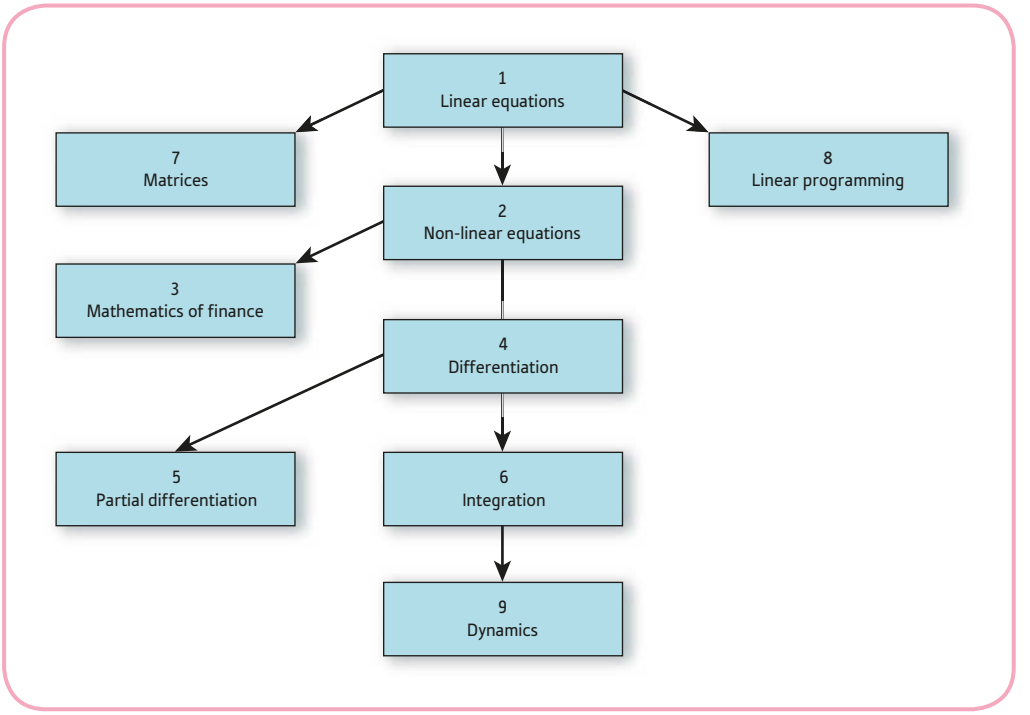
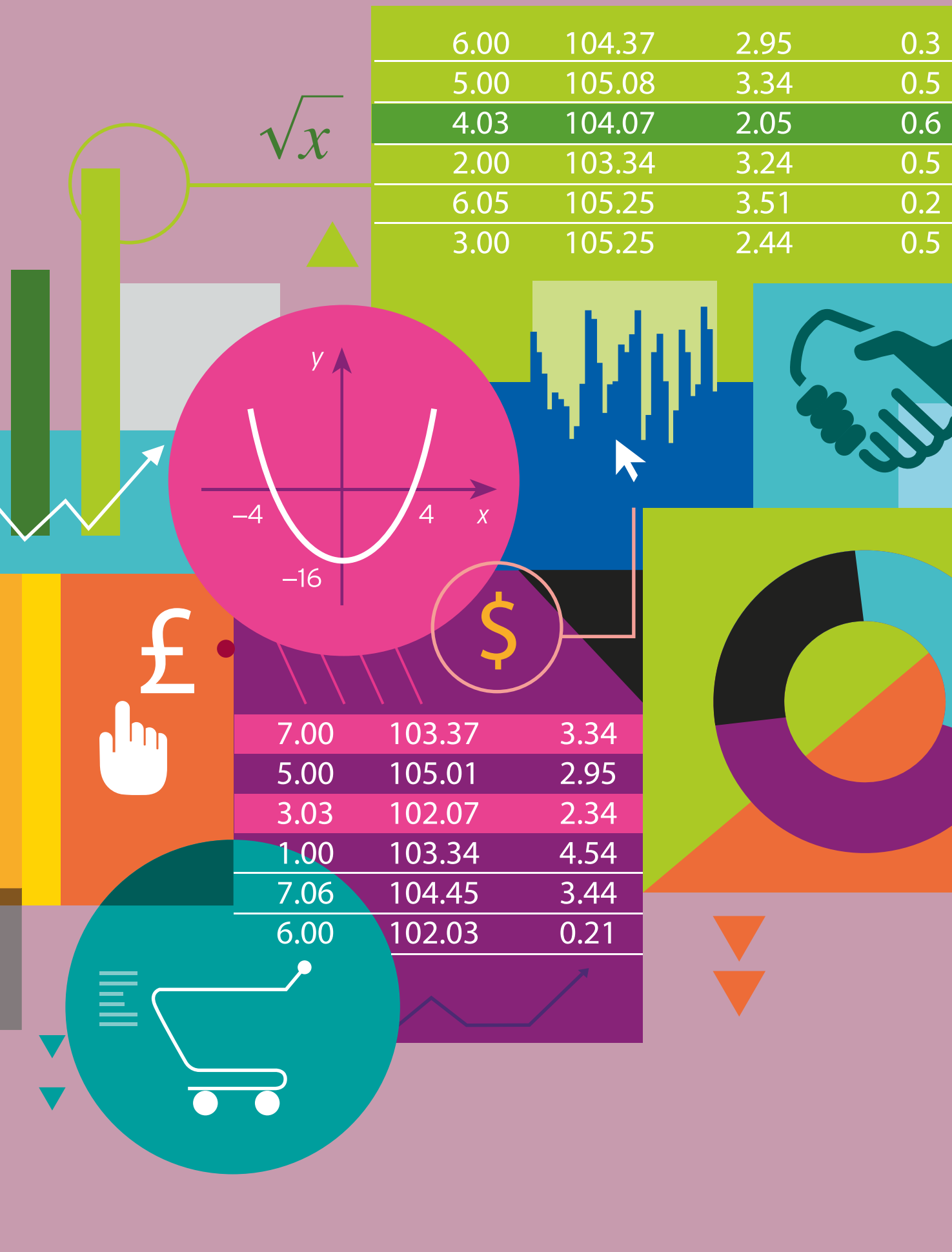


Figure I.1

are provided at the back of the book so that you can check to see how well you have done. If you do get any of the questions wrong, it would be worth re-doing that question perhaps writing down full working so that you can spot your mistake more easily. The final section contains several examination-style problems which are more challenging. They tend to be longer than the questions encountered so far in the exercises and require more confidence and experience. We recommend that you leave these until the end of the course and use them in your build-up to the final exams.

I hope that this text helps you to succeed in your mathematics course. You never know, you might even enjoy it. Remember to wear your engineer's hat while reading the text. I have done my best to make the material as accessible as possible. The rest is up to you!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CHAPTER 1
Linear Equations

The main aim of this chapter is to introduce the mathematics of linear equations. This is an obvious first choice in an introductory text, since it is an easy topic which has many applications. There are seven sections, which are intended to be read in the order that they appear.

Sections 1.1, 1.2, 1.3, 1.4 and 1.6 are devoted to mathematical methods. They serve to revise the rules of arithmetic and algebra, which you probably met at school but may have forgotten. In particular, the properties of negative numbers and fractions are considered. A reminder is given on how to multiply out brackets and how to manipulate mathematical expressions. You are also shown how to solve simultaneous linear equations. Systems of two equations in two unknowns can be solved using graphs, which are described in Section 1.3. However, the preferred method uses elimination, which is considered in Section 1.4. This algebraic approach has the advantage that it always gives an exact solution and it extends readily to larger systems of equations.

The remaining two sections are reserved for applications in microeconomics and macroeconomics. You may be pleasantly surprised by how much economic theory you can analyse using just the basic mathematical tools considered here. Section 1.5 introduces the fundamental concept of an economic function and describes how to calculate equilibrium prices and quantities in supply and demand theory. Section 1.7 deals with national income determination in simple macroeconomic models.

The first six sections underpin the rest of the text and are essential reading. The final section is not quite as important and can be omitted at this stage.

SECTION 1.1
Introduction to algebra

Objectives

At the end of this section you should be able to:

- Add, subtract, multiply and divide negative numbers.
- Understand what is meant by an algebraic expression.
- Evaluate algebraic expressions numerically.
- Simplify algebraic expressions by collecting like terms.
- Multiply out brackets.
- Factorise algebraic expressions.

ALGEBRA IS BORING

There is no getting away from the fact that algebra *is* boring. Doubtless there are a few enthusiasts who get a kick out of algebraic manipulation, but economics and business students are rarely to be found in this category. Indeed, the mere mention of the word 'algebra' is enough to strike fear into the heart of many a first-year student. Unfortunately, you cannot get very far with mathematics unless you have completely mastered this topic. An apposite analogy is the game of chess. Before you can begin to play a game of chess, it is necessary to go through the tedium of learning the moves of individual pieces. In the same way it is essential that you learn the rules of algebra before you can enjoy the 'game' of mathematics. Of course, just because you know the rules does not mean that you are going to excel at the game, and no one is expecting you to become a grandmaster of mathematics. However, you should at least be able to follow the mathematics presented in economics books and journals as well as to solve simple problems for yourself.

Advice

If you have studied mathematics recently, then you will find the material in the first few sections of the text fairly straightforward. You may prefer just to try the questions in the starred exercise at the end of each section to get yourself back up to speed. However, if it has been some time since you have studied this subject, our advice is very different. Please work through the material thoroughly even if it is vaguely familiar. Make sure that you do the problems as they arise, checking your answers with those provided at the back of this text. The material has been broken down into three subsections:

- negative numbers;
- expressions;
- brackets.

You might like to work through these subsections on separate occasions to enable the ideas to sink in. To rush this topic now is likely to give you only a half-baked understanding, which will result in hours of frustration when you study the later chapters of this text.

1.1.1 Negative numbers

In mathematics numbers are classified into one of three types: positive, negative or zero. At school you were probably introduced to the idea of a negative number via the temperature on a thermometer scale measured in degrees centigrade. A number such as -5 would then be interpreted as a temperature of 5 degrees below freezing. In personal finance a negative bank balance would indicate that an account is 'in the red' or 'in debit'. Similarly, a firm's profit of $-500\,000$ signifies a loss of half a million.

The rules for the multiplication of negative numbers are

$$\text{negative} \times \text{negative} = \text{positive}$$

$$\text{negative} \times \text{positive} = \text{negative}$$

It does not matter in which order two numbers are multiplied, so

$$\text{positive} \times \text{negative} = \text{negative}$$

These rules produce, respectively,

$$(-2) \times (-3) = 6$$

$$(-4) \times 5 = -20$$

$$7 \times (-5) = -35$$

Also, because division is the same sort of operation as multiplication (it just undoes the result of multiplication and takes you back to where you started), exactly the same rules apply when one number is divided by another. For example,

$$(-15) \div (-3) = 5$$

$$(-16) \div 2 = -8$$

$$2 \div (-4) = -1/2$$

In general, to multiply or divide lots of numbers it is probably simplest to ignore the signs to begin with and just to work the answer out. The final result is negative if the total number of minus signs is odd and positive if the total number is even.

Example

Evaluate

(a) $(-2) \times (-4) \times (-1) \times 2 \times (-1) \times (-3)$ **(b)** $\dfrac{5 \times (-4) \times (-1) \times (-3)}{(-6) \times 2}$

Solution

(a) Ignoring the signs gives

$$2 \times 4 \times 1 \times 2 \times 1 \times 3 = 48$$

There are an odd number of minus signs (in fact, five), so the answer is -48.

(b) Ignoring the signs gives

$$\frac{5 \times 4 \times 1 \times 3}{6 \times 2} = \frac{60}{12} = 5$$

There are an even number of minus signs (in fact, four), so the answer is 5.

Advice

Attempt the following problem yourself both with and without a calculator. On most machines a negative number such as -6 is entered by pressing the button labelled $(-)$ followed by 6.

Practice Problem

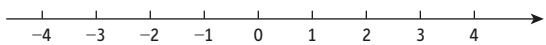
1. **(1)** Without using a calculator, evaluate

(a) $5 \times (-6)$ **(b)** $(-1) \times (-2)$ **(c)** $(-50) \div 10$

(d) $(-5) \div (-1)$ **(e)** $2 \times (-1) \times (-3) \times 6$ **(f)** $\dfrac{2 \times (-1) \times (-3) \times 6}{(-2) \times 3 \times 6}$

(2) Confirm your answer to part (1) using a calculator.

To add or subtract negative numbers it helps to think in terms of a number line:



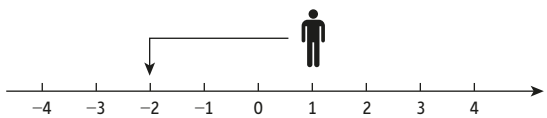
If b is a positive number, then

$$a - b$$

can be thought of as an instruction to start at a and to move b units to the left. For example,

$$1 - 3 = -2$$

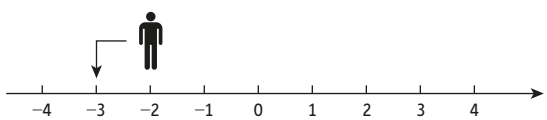
because if you start at 1 and move 3 units to the left, you end up at -2:



Similarly,

$$-2 - 1 = -3$$

because 1 unit to the left of -2 is -3.



On the other hand,

$$a - (-b)$$

is taken to be $a + b$. This follows from the rule for multiplying two negative numbers, since

$$-(-b) = (-1) \times (-b) = b$$

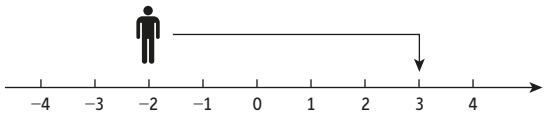
Consequently, to evaluate

$$a - (-b)$$

you start at a and move b units to the right (that is, in the positive direction). For example,

$$-2 - (-5) = -2 + 5 = 3$$

because if you start at -2 and move 5 units to the right, you end up at 3.



Practice Problem

2. **(1)** Without using a calculator, evaluate

(a) $1 - 2$ **(b)** $-3 - 4$ **(c)** $1 - (-4)$

(d) $-1 - (-1)$ **(e)** $-72 - 19$ **(f)** $-53 - (-48)$

(2) Confirm your answer to part (1) using a calculator.

1.1.2 Expressions

In algebra, letters are used to represent numbers. In pure mathematics the most common letters used are x and y. However, in applications it is helpful to choose letters that are more meaningful, so we might use Q for quantity and I for investment. An algebraic expression is then simply a combination of these letters, brackets and other mathematical symbols such as $+$ or $-$. For example, the expression

$$P\left(1 + \frac{r}{100}\right)^n$$

can be used to work out how money in a savings account grows over a period of time. The letters P, r and n represent the original sum invested (called the principal – hence the use of the letter P), the rate of interest and the number of years, respectively. To work it all out, you not only need to replace these letters by actual numbers, but you also need to understand the various conventions that go with algebraic expressions such as this.

In algebra, when we multiply two numbers represented by letters, we usually suppress the multiplication sign between them. The product of a and b would simply be written as ab without bothering to put the multiplication sign between the symbols. Likewise, when a number represented by the letter Y is doubled, we write $2Y$. In this case we not only suppress the multiplication sign but adopt the convention of writing the number in front of the letter. Here are some further examples:

$P \times Q$ is written as PQ

$d \times 8$ is written as $8d$

$n \times 6 \times t$ is written as $6nt$

$z \times z$ is written as z^2 (using the index 2 to indicate squaring a number)

$1 \times t$ is written as t (since multiplying by 1 does not change a number)